

Degeneration of Rational points on an Elliptic Curve and the Coefficients of the Laurent Expansion of an A_2 -model

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Abstract

We were interested in an A_2 -model of an elliptic curve. It was used to solve Fermat's Last Theorem. We investigated the expansion coefficients of rational points on a degenerating A_2 -model by focusing on the structure of rational points on an A_2 -model. As a result, we found that the expansion coefficients of mP have a regular pattern depending on the remainder of m modulo 3.

1 Introduction

In the field of mathematics, long-standing research has focused on questions such as the number and structure of rational points on curves. We were interested in an A_2 -model, an elliptic curve. An A_2 -model ($y^2 = x^3 - x + 1/s^2$) degenerates as s approaches 0. In this study, we focused on the structure of rational points on an A_2 -model and investigated the expansion coefficients of rational points on a degenerating A_2 -model.

2 Theory and Experiment

First, we expanded the coordinates of rational points on a degenerating A_2 -model and found a regular pattern depending on the remainder of m modulo 3.

Next, we expressed the expansion coefficients of the first term and the second term of x and y -coordinates of mP by recurrence relations.

Finally, we solved recurrence relations and determined the expansion coefficients.

3 Results

In case of $m \equiv 1 \pmod{3}$,

$$\left[m + \frac{(m-1)(3m^3 + 3m^2 - 3m - 2)}{24} s^2 + \dots, \frac{1}{s} + \frac{(m-1)m(m+1)}{2} s + \dots \right]$$

In case of $m \equiv 2 \pmod{3}$,

$$\left[-m + \frac{(m+1)(3m^3 - 3m^2 - 3m + 2)}{24} s^2 + \dots, -\frac{1}{s} + \frac{(m-1)m(m+1)}{2} s + \dots \right]$$

In case of $m \equiv 0 \pmod{3}$,

$$\left[\left(\frac{2}{m}\right)^2 \frac{1}{s^2} - \frac{1}{3m^2} + \dots, \left(-\frac{2}{m}\right)^3 \frac{1}{s^3} + \frac{1}{m^3} \frac{1}{s} + \dots \right]$$

We found that the expansion coefficients of the first term and second term of x and y -coordinates of mP have a regular pattern depending on the remainder of m modulo 3.

We hypothesize the expansion coefficients of other terms of mP have a similar regular pattern.

4 Conclusion

We found the coefficients of the Laurent expansion of the first and second terms of mP on the A_2 -model. In future work, we would like to research the coefficients of the Laurent expansion of rational points on other elliptic curves and possible applications to other fields, such as physics.

5 References

J.H. Silverman, J.Tate.(1992). Rational Points on Elliptic Curves. Springer-Verlag.